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การประยุกต์ Relative Order ด้วยอนุพันธ์ Lie เพื่อคัดเลือกรูปแบบการควบคุม  
Application of Relative Order under Lie Derivative  
to Classify the Control Configurations

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สำนักพัฒนาบัณฑิตศึกษาและวิจัยด้านวิทยาศาสตร์และเทคโนโลยี (สบว)

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#### บทคัดย่อ

ทฤษฎีความสามารถในการควบคุมได้เชิงโครงสร้างสามารถประเมินรูปแบบการควบคุมต่างๆ โดยใช้ความสัมพันธ์ของตัวแปรที่อยู่ในกระบวนการ งานวิจัยนี้ได้ทำการทดสอบต้นแบบการสร้างระบบควบคุมอัตโนมัติซึ่งใช้ความสามารถในการควบคุมได้เชิงโครงสร้างเป็นพื้นฐาน การวิเคราะห์ตรรกะการเชื่อมต่อ (DI) ถูกนำมาประยุกต์ใช้ และใส่ลงไปในตัวแบบการควบคุมนี้เพื่อพัฒนากระบวนการทำงาน เทคนิคนี้ใช้พื้นฐานเดียวกับต้นแบบจึงทำให้สามารถทำงานร่วมกันได้ ดังนั้นอาจกล่าวได้ว่าตรรกะการเชื่อมต้อมีบทบาทในการทำงานสนับสนุนดีกรีความสัมพันธ์ในกรณีที่ดีกรีความสัมพันธ์ไม่สามารถใช้บังคับรูปแบบการควบคุมที่ดีที่สุดได้ กรณีศึกษาของโรงงานแปรรูปแอมโมเนียพบว่าได้ผลการทดสอบเป็นที่น่าพอใจ การวิเคราะห์ตรรกะการเชื่อมต่อจะเข้าไปมีบทบาท 29 block แบ่งได้ดังนี้ ตรรกะการเชื่อมต้อมีบทบาทเต็ม 5 block ตรรกะการเชื่อมต้อมีบทบาทพอสมควร 8 block ตรรกะการเชื่อมต้อมีบทบาท 8 block และตรรกะการเชื่อมต่อที่ไม่มีค่าสำหรับการเลือกอีก 16 block

**คำสำคัญ :** ทฤษฎีความสามารถในการควบคุมได้เชิงโครงสร้าง ตรรกะการเชื่อมต่อ ตารางอัตราขยายสัมพันธ์

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## Abstract

The structural controllability evaluates alternative control configurations by the relationship of process variables. This article focuses on the test of the prototype of an automatic control system based on structural analysis. An algorithm named DI analysis is applied to select the control configurations with the same relative order. It plays a supplementary role when the relative order fails to indicate the best control configuration. A case study of the pilot scale of monazite processing plant gives a good agreements. 29 blocks are investigated with DI analysis. We can nominate the best control structure from 5 blocks. The number of possible configurations can be reduced in 8 blocks. And it does not have any valuable value (DI value) for screening the last 16 blocks.

**Keywords :** structural controllability, decoupling index, relative gain array

## 1. Introduction

Normally, purposes of process design systems are focussed on process stability, operability, and controllability. Controllability is usually considered after process design is completed and after requires redesign. In recent years, performance of controllability for process design, which requires knowledge of the relationship between the input and output variables, is more concentrated especially on more complicated operating systems (Srinophakun, 1996).

For multiple-input/multiple-output (MIMO) control systems, there are multiple controlled objectives. In such processes, each control objective can be considered separately from the others as long as they do not interact with each other, but this case is almost impossible. Pairing the controlled and manipulated variables to minimize the effect of interaction is very important.

There are several methods to deal with this problem; Relative Gain Array (RGA) and Niederlinski Index (NI) are powerful tools used to address such problem. These methods named "Quantity method" are very complicated and require much information which is usually missing or unknown in the early design stage. They are obtained from performing dynamic simulation of target process. Besides, they may be unreliable and impossible especially for large-scale systems. Hence, the structural controllability analysis is proposed and tried.

Because of the complicated of the system controllability analyses many studies have tried to solve this problem. Dautidis and Kravaris introduced guidelines for the structural evaluation of alternative control configurations based on the structural controllability (Dautidis & Kravaris, 1992). Then, Srinophakun used digraph theory, cause and effect matrix to build a prototype of automatic control system (Srinophakun, 1996). It is used to find the best control configuration for control system design usually used in an early stage of process design. After that, Namkang developed this model and tested with a Monazite processing plant (Namkang, 2000). This

work focuses on applying a technique in the prototype of automatic control system and test with case studies. A relative order technique used to indicate the best control configuration existing in the prototype is developed.

## 2. Decoupling Index (DI)

As described in the previous section, the prototype of automatic control is used to achieve this aim. It can be used to design control system at an early stage and should be used simultaneously with process design for controllability. This prototype picturized all of process variables in whole plant. All process variables are included together to consider their relationships using the state-space models of each equipment in each unit. Then the cause and effect matrix (CEM) is generated from the relationships of process variables. The rank of CEM will be checked using output set theory. If there is a rank deficiency, it can be modified by eliminating the non-critical objective in such CEM or redesign (Srinophakun, 1996 and Namkang, 2000). The defective structures are determined for controllability. The variables in the same stream of each unit are merged together to reduce the number of variables. If there are relationships between the equipment, they are also connected together. Then, the plant CEM is obtained from assembling all CEM in each unit together. It is reordered into diagonal form. When the procedure comes up to this step, the methods to evaluate the control configurations are applied. The relative order technique existing in this prototype is considered representing in relative order matrix, which each element is a relative order of output  $y_i$  with respect to manipulated input  $u_j$  in such configuration. It indicates how much a system is decoupled. The best configuration is the sum of relative order in diagonal form the smallest value.

In some cases, there are more than one control configurations, which are the same smallest relative order

in diagonal form. Next step, the heuristic guidelines are used to indicate the best one by users (Srinophakun, 1996). It is interesting to apply an algorithm used to indicate the best control configuration more precisely before using the heuristic guidelines as the last mission.

The algorithm is introduced to screen the better control configuration if relative order values cannot identify the best case (Lee et al., 2001). DI analysis is the additional tools used to indicate the better control configurations. Besides, it gives results faster. The physical meaning of this concept is the relation of manipulated and objective variables. It uses the same direction compared with relative gain array. DI is calculated as follow.

$$DI = \sum_i^n \sum_{j \neq i}^m \frac{r_i}{r_{ij}} \quad (1)$$

Where  $r_{ij} \equiv$  Relative order of the output  $y_j$  with respect to a manipulated input  $u_j$

$n \equiv$  number of output variable

$m \equiv$  number of manipulated variable

$r_i \equiv \min(r_{i1}, r_{i2}, \dots, r_{im})$

The relative order matrix with the smallest DI is selected as the best control configuration. To illustrate about DI, an example is shown in figure 1.

$$\begin{bmatrix} 1 & 4 & 5 \\ 10 & 1 & 2 \\ 8 & 4 & 2 \end{bmatrix}$$

$$DI = \frac{1}{4} + \frac{1}{5} + \frac{1}{10} + \frac{1}{2} + \frac{2}{8} + \frac{2}{4} = 1.8$$

$$\begin{bmatrix} 1 & 4 & 3 \\ 4 & 1 & 5 \\ 3 & 3 & 2 \end{bmatrix}$$

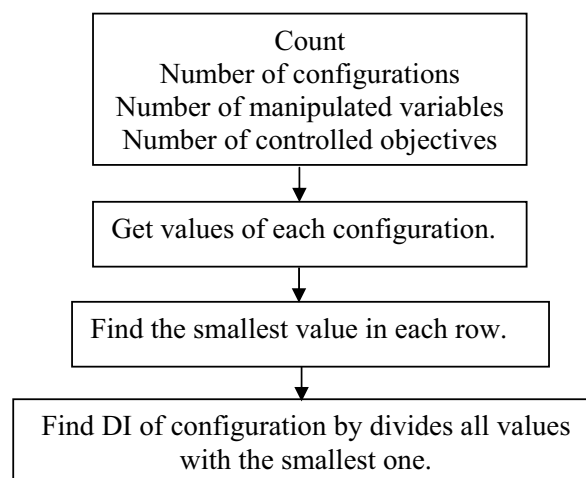
$$DI = \frac{1}{4} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{2}{3} + \frac{2}{3} = 2.366$$

**Figure 1** Example of DI analysis

Both configurations as shown in Figure 1 have the same relative order in diagonal form that is 4. When DI is calculated, DI in configuration 1 and 2 are 1.8 and 2.366 respectively. The best one with the smallest DI is configuration number 1.

### 3. DI code

The DI analysis is applied in the prototype to address the issue of more robust performance. This DI analysis is coded with Microsoft Visual C++ version 6.0 and applied to make it ease for selecting the most favorable control configuration. From the definition of DI, the flowchart generates as following steps are shown in Figure 2.



**Figure 2** Algorithm to code DI analysis

There are four main steps in this analysis. First, a minimum element in each row is provided. Then, DI in each row is calculated, and next, summed together to obtain DI of each configuration. Finally the minimum DI is determined.

To apply this code in the prototype of automatic control system, it is used as subroutine of the prototype. All critical variables in DI subroutine are designed to match with the current output of control configuration. The CEM from the previous version provides information is an input to the DI part. The combination of DI analysis provides the prototype capability to indicate the control configurations from the relative order analysis stages.

In conclusion, heuristic to indicate the control configurations of this prototype involves 3 steps. Relative order matrixes are found first if there are many alternatives. Then, this data is dealt with using the DI analysis. By now, designer can indicate the most approving control configuration. If the designer faces the difficulty to nominate the best one, the special guidelines of selection play the vital role at this last stage.

#### 4. Case study

A case study of pilot scale monazite processing plant is used for the prototype with DI analysis. It is a production process of rare earth from monazite. At the beginning, there is no control system. Namkang designed

control system of this plant using the prototype of automatic control system (Namkang, 2000). Now, the DI analysis is proposed to support those steps, which is applied in this prototype.

There are 19 from 22 units that control systems are designed in the pilot scale monazite processing plant. Three remaining units are used as chemical station for chemical preparation and chemical storage, which are unit 5, unit 11 and unit 20. So, it is not necessary to have control system design.

In this work, DI code is tested in the case of many feasible control configurations occurred only. There are 12 units in this plant; unit1-1, unit 1, unit 2, unit 3, unit 8, unit 9, unit 12, unit 13, unit 15, unit 16, unit 17 and unit 19.

For case study tested, first step, mass and energy balances in each equipment are used to perform state equations and output equations, then this data is fed to the prototype (Injarean, 2000). From system pairing step, if there are many relationships in many variables, they are separated in many blocks. Then, each block is focused to evaluate the control configurations. Finally, the relative order matrix of each unit is obtained. For 12 units tested there are many control configurations which have the smallest relative order in diagonal form. DI is calculated in next step, the results are shown in figure 3 using unit number 1 as an example.

Unit number1: Digesting Unit

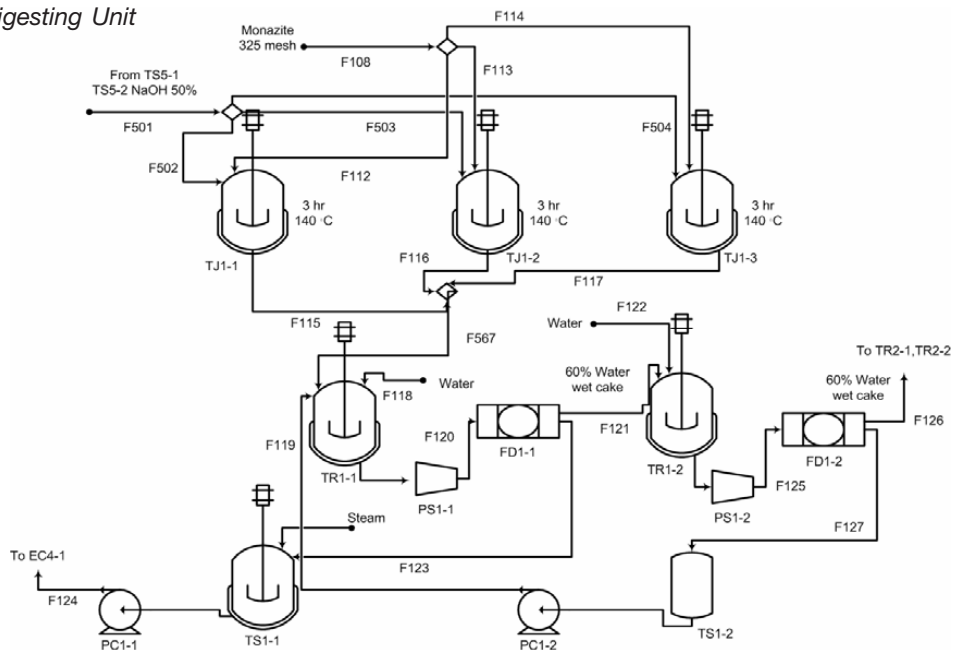


Figure 3 Flow sheet of digesting unit (Unit number1)

The monazite from grinding unit is reacted with 50% by weight concentration of NaOH, which came from acid alkaline station in the digest tank. The compounds,  $\text{Na}_2\text{U}_2\text{O}_7$ ,  $\text{Th}(\text{OH})_4$ ,  $\text{Ce}(\text{OH})_4$  and  $\text{Re}(\text{OH})_3$ , can be produced from this reaction. Then, these compounds are feed to precipitate washing tank or dilute solution tank and filter, respectively. The liquid from filter is  $\text{Na}_3\text{PO}_4$ , which is passed to tri-sodium phosphate production unit and cake of uranium, thorium and light and heavy rare earth in hydroxide form ( $\text{OH}^-$ ) are sent to dissolution unit. There are five components of main equipment in this unit.

- Reactor (TJ1-1, TJ1-2, TJ1-3)
- Mixer (Mixer 1 )
- Washed and dilution mixture tank (TR1-1, TR2-2)
- Filter (FD1-1, FD1-2)
- Storage tank (TS1-1, TS1-2)

The state equations and output equations are generated first using mass and energy balance. From Namkang's work in 2000, there are four blocks in system pairing step considered that are blocks 7, 9, 11 and 13 as shown in Figures 4, 5, 6 and 7.

Block 7

	F502	F112	F115
$r_{j11}$	x	x	
$V_{j11}$	x	x	x
$t_{j11}$	x	x	x

Configuration number 1.

	F502	F112	F115
$r_{j11}$	1	1	
$V_{j11}$	1	1	1
$t_{j11}$	1	1	1

Configuration number 2.

	F502	F115	F112
$r_{j11}$	1		1
$V_{j11}$	1	1	1
$t_{j11}$	1	1	1

Configuration number 3.

	F112	F502	F115
$r_{j11}$	1	1	
$V_{j11}$	1	1	1
$t_{j11}$	1	1	1

Configuration number 4.

	F112	F115	F502
$r_{j11}$	1		1
$V_{j11}$	1	1	1
$t_{j11}$	1	1	1

Figure 4 Relative order matrix of block 7

Block 9

	F503	F113	F116
$r_{j12}$	x	x	
$V_{j12}$	x	x	x
$t_{j12}$	x	x	x

Configuration number 1.

	F503	F113	F116
$r_{j12}$	1	1	
$V_{j12}$	1	1	1
$t_{j12}$	1	1	1

Configuration number 2.

	F503	F116	F113
$r_{j12}$	1		1
$V_{j12}$	1	1	1
$t_{j12}$	1	1	1

Configuration number 3.

	F113	F503	F116
$r_{j12}$	1	1	
$V_{j12}$	1	1	1
$t_{j12}$	1	1	1

Configuration number 4.

	F113	F116	F503
$r_{j12}$	1		1
$V_{j12}$	1	1	1
$t_{j12}$	1	1	1

Figure 5 Relative order matrix of block 9

Block 11

	F504	F114	F117
$r_{j13}$	x	x	
$V_{j13}$	x	x	x
$t_{j13}$	x	x	x

Configuration number 1.

	F504	F114	F117
$r_{j13}$	1	1	
$V_{j13}$	1	1	1
$t_{j13}$	1	1	1

Configuration number 2.

	F504	F117	F114
$r_{j13}$	1		1
$V_{j13}$	1	1	1
$t_{j13}$	1	1	1

Configuration number 3.

	F114	F504	F117
$r_{j13}$	1	1	
$V_{j13}$	1	1	1
$t_{j13}$	1	1	1

Configuration number 4.

	F114	F117	F504
$r_{j13}$	1		1
$V_{j13}$	1	1	1
$t_{j13}$	1	1	1

**Figure 6** Relative order matrix of block 11

Block 13

	F120	F122	F118
$V_{r12}$	x	x	
$V_{r11}$	x		x

Configuration number 1.

	F120	F118
$V_{r12}$	1	
$V_{r11}$	1	1

Configuration number 2.

	F122	F120
$V_{r12}$	1	1
$V_{r11}$		1

Configuration number 3.

	F122	F118
$V_{r12}$	1	
$V_{r11}$		1

**Figure 7** Relative order matrix of block 13

DIs of all configurations are shown in Table 1.

**Table 1** DI of all configurations in each block of unit number 1

Block	DI of Configuration			
	1	2	3	4
7	5	5	5	5
9	5	5	5	5
11	5	5	5	5
13	1	1	0	-

From Table 1, for block 7 there are 4 configurations, and all have the smallest DI. The user knowledge are used to select the best one again as same as the other blocks. The results are shown below in Table 2.

**Table 2** Details of configurations selected and guidelines used of unit number 1

Block	No. of configuration	Smallest DI (configuration)	Selected configuration No.	Guidelines No. used
7	4	4	3	5, 7
9	4	4	3	5, 7
11	4	4	3	5, 7
13	3	1	3	-

## 5. Discussion

In this work, the prototype with DI analysis is used for the plant-wide scale with many feasible design alternatives of Namkang's work. DI analysis offers some advantages.

It is found that DI analyses the control configurations and classified the roles in 3 areas as follow,

1. *DI works fully.* It was used to indicate one control configuration as the best control configuration rapidly without any user knowledge.

2. *DI works fairly well.* DI can be used to screen some control configurations, which have the higher DI only. It stated that DI reduced the design alternatives. So, selecting the best control configuration was more accurate based on user knowledge.

3. *DI idles.* DI cannot help anything. Because the relative order in each element of all alternatives is the same, the same DIs appear. The user knowledge to indicate the best one is necessary in this selection.

For DI analysis, if the number of manipulated variables is more than the number of controlled objectives, the arrays of manipulated variables in each control configuration are differently arranged. In this case, each DI analysis shows the different values if there are the variety of relative orders in each element and, therefore, the best DI exists. This considers that DI can be used to select the control configurations after perform the relative order analysis.

In addition, it is obvious that the relative order in each design alternatives are almost the same values. DI analysis has the roles smaller than possible in some cases. These should come from the system, which has a low order. So, what system is appropriate with this analysis should be investigated. The possible system should be the one with higher order. If the system has the higher order, DI has more potential selectivity. The number of paths between these process variables are therefore increased and complex.

## 6. Conclusion

In this work, the evaluation of control configuration of process variables is proposed based on the structural controllability. The prototype of automatic control system is introduced to find the best control configuration at an early stage of control system design. This work focused on the prototype modification, the algorithm is proposed and applied to make the prototype more robust. The concept is DI analysis by Lee et al in 2001. They proved this analysis could be used to screen the design alternatives of control configurations. It could reduce the number of the design alternatives. So the reliance of the selection gains more. The DI analysis was applied into the prototype using Microsoft Visual C++ version 6.0. The pilot scale monazite processing plant is applied to validate DI analysis.

There are 29 blocks, which have many design alternatives, in the pilot scale monazite processing plant test. DI works fully 5 blocks, fairly well 8 blocks and idles 16 blocks.

However, this analysis worked based on the structural controllability. It can be associated with relative order, which was the tool to evaluate the control configurations that existed previously. In addition, the structural controllability is appropriate to design the control system at the beginning of project, in which the dynamic simulation is not performed and the information required is not known.

## 7. Acknowledgement

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