AB-ไอดีลแบบแอนติวิภัชนัย

Anti-Fuzzy AB-Ideals

ลี ศาสนพิทักษ์ และ บุญยงค์ ศรีพลแผ้ว*

Lee Sassanapitax and Boonyong Sriponpaew*

ภาควิชาคณิตศาสตร์ คณะวิทยาศาสตร์ มหาวิทยาลัยบูรพา

Mathematics Department, Faculty of Science, Burapha University

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บทคัดย่อ

ในบทความนี้ แนวคิดของไอดีลแบบแอนติวิภัชนัยของพีชคณิตแบบ AB ได้รับการแนะนำ และผลลัพธ์ที่เกี่ยวข้องได้รับการพิสูจน์ อีกทั้งเราได้ศึกษาลักษณะเฉพาะโดยใช้แนวคิดของสับเซตลำดับชั้นที่ต่ำกว่า ยิ่งไปกว่านั้นเราได้ให้คุณสมบัติของผลคูณคาร์ทีเซียนของไอดีลแบบแอนติวิภัชนัย

คำสำคัญ : พีชคณิตแบบ AB ; ไอดีลแบบวิภัชนัย ; ไอดีลแบบแอนติวิภัชนัย

Abstract

In this paper, the concepts of anti-fuzzy AB-ideals of AB-algebras are introduced and some related results are proved. We also study their characterizations by using the concept of lower level subset. Furthermore, we provide the properties of the Cartesian product of anti-fuzzy AB-ideals.

Keywords : AB-algebras ; fuzzy ideals ; anti-fuzzy ideals

*Corresponding author. E-mail : boonyong@buu.ac.th

Introduction

The concept of "Fuzzy Sets" was first introduced in (Zadeh, 1965) as a set X with associated function mapping X into interval $[0,1]$. He investigated some basic properties and operations on this fuzzy sets. Based on his works, Fuzzy set theory has been quickly developed in many perspectives by many mathematicians on any spaces such as groups, rings, semigroups and topological spaces. Theory of fuzzy sets can be broadly applied in various fields such as facial recognition, medical diagnosis, machine learning, design of robots, computer simulation and engineering planning (see (Ahmadi *et al.*, 2018), (Bajec, 2005), (Chen *et al.*, 2017), (Hüllermeier, 2015), (Kumar, 2013), (Pusztai *et al.*, 2019), (Lee *et al.*, 1996)). The concept of fuzzy sets in the area of group theory was first initiated in (Rosenfeld, 1971). Since then the concept of fuzzy set in abstract algebra was widely extended into many results. Recently, (Hameed *et al.*, 2017) proposed an algebraic structure called AB-algebra and (Hameed *et al.*, 2018) presented fuzzy AB-ideals and their relevant properties. From this fuzzy setting, it is natural to study anti-fuzzy sets in AB-algebra. In this paper, we introduce the notion of anti-fuzzy ideals in AB-algebras and prove some related properties including their t-level sets, Cartesian products and homomorphism of anti-fuzzy ideals.

Methods

Firstly, let us recall definitions of AB-algebras, its ideals, fuzzy AB-ideals and anti-fuzzy AB-ideals as well as anti-fuzzy AB-subalgebras as follows.

Definition 1 (Hameed *et al*., 2017) An *AB-algebra* $(X,*,0)$ is an algebra of type $(2,0)$, i.e. X is a nonempty set, $*$ is a binary operation on X and 0 is the constant element in X satisfying the following axioms:

(AB_1) $((x*y)*(z*y))*(x*z)=0$,

(AB_2) $0*x=0$,

(AB_3) $x*0=x$,

for all $x,y,z\in X$.

A binary relation $\leq$ on an AB-algebra X was defined as follows: for all $x,y\in X$,

$x\leq y$ if and only if $x*y=0$.

Definition 2 (Hameed *et al*., 2017) A nonempty subset I of an AB-algebra X is called an *AB-ideal* of X if it satisfies the following conditions:

(ABI_1) $0\in I$,

(ABI_2) for each $x,y,z\in X$, $(x*y)*z\in I$ and $y\in I$ imply $x*z\in I$.

Definition 3 (Zadeh, 1965) A *fuzzy set* in a nonempty set X is a function $f : X \to [0,1]$.

We call such function a *fuzzy subset* of X

Definition 4 (Hameed *et al*., 2018) A fuzzy set f in an AB-algebra X is called an *fuzzy AB-ideal* of X if it satisfies the following properties: for any $x, y, z \in X$,

1. $f(0) \geq f(x)$,
2. $f(x * z) \geq \min\{f\big((x * y) * z\big), f(y)\}$.

To further the study of fuzzy set theory in AB-algebras, we introduce the concept of an anti-fuzzy AB-ideal as follows.

Definition 5 A fuzzy set f in an AB-algebra X is called an *anti-fuzzy AB-ideal* of X if it satisfies the following properties: for any $x, y, z \in X$,

1. $f(0) \leq f(x)$,
2. $f(x * z) \leq \max\{f\big((x * y) * z\big), f(y)\}$

Definition 6 A fuzzy set f in an AB-algebra X is called an *anti-fuzzy AB-subalgebra* of X if for any $x, y \in X$
,

$$f(x * y) \leq \max\{f(x), f(y)\}.$$

Results

In this section, we investigate some properties of anti-fuzzy AB-ideals and anti-fuzzy AB-subalgebras.

Theorem 1 Let f be an anti-fuzzy AB-ideal of an AB-algebra $(X, *, 0)$ and $x, y, z \in X$.

If $z * y \leq x$, then $f(z) \leq \max\{f(x), f(y)\}$.

Proof Since $f(z) = f(z * 0) \leq \max\{f\big((z * x) * 0\big), f(x)\} = \max\{f(z * x), f(x)\}$ and $f(z * x) \leq \max\{f\big((z * y) * x\big), f(y)\} = \max\{f(0), f(y)\} = f(y)$, it can be concluded that $f(z) \leq \max\{f(x), f(y)\}$. □

The following result shows that an anti-fuzzy AB-ideal preserves the order $\leq$.

Corollary 1 Let f be an anti-fuzzy AB-ideal of an AB-algebra $(X, *, 0)$ and $x, y \in X$.

If $x \leq y$, then $f(x) \leq f(y)$.

Proof Since X is AB-algebra and by assumption, we have $x * 0 = x \leq y$.

By Theorem 1, $f(x) \leq \max\{f(y), f(0)\} = f(y)$. □

Theorem 2 Every anti-fuzzy AB-ideal of an AB-algebra $(X, *, 0)$ is an anti-fuzzy AB-subalgebra of X.

Proof Let f be an anti-fuzzy AB-ideal of an AB-algebra of X and $x, y \in X$. Then

$$\begin{aligned} f(x*y) &\le \max\{f((x*x)*y), f(x)\} \\ &= \max\{f(0*y), f(x)\} \\ &= \max\{f(0), f(x)\} \\ &= f(x) \\ &\le \max\{f(y), f(x)\}. \end{aligned}$$

□

Definition 7 Let $t \in [0,1]$ and let α be a fuzzy set of a nonempty set X. Then $L(\alpha, t) = \{x \in X : \alpha(x) \le t\}$ is called *lower* t*-level* of α.

The following theorems provide characterizations of anti-fuzzy ideals in AB-algebras.

Theorem 3 Let $(X, *, 0)$ be an AB-algebra and α a fuzzy set in X. Then α is an anti-fuzzy AB-ideal of X if and only if $L(\alpha, t) \ne \varnothing$ implies $L(\alpha, t)$ is an AB-ideal of X.

Proof Assume α is an anti-fuzzy AB-ideal of X. Let $t \in [0,1]$ be such that $L(\alpha, t) \ne \varnothing$. Then there exists $x \in L(\alpha, t)$. Since $\alpha(0) \le \alpha(x) \le t$, we have $0 \in L(\alpha, t)$. Let $(x*y)*z \in L(\alpha, t)$ and $y \in L(\alpha, t)$. Then $\alpha((x*y)*z) \le t$ and $\alpha(y) \le t$. Since $\alpha(x*z) \le \max\{\alpha((x*y)*z), \alpha(y)\} \le t$, it follows that $x*z \in L(\alpha, t)$.

To show the sufficiency part, we suppose on the contrary that $\alpha(0) > \alpha(x)$ for some $x \in X$. Take $t_1 = \dfrac{\alpha(0) + \alpha(x)}{2} \in [0,1]$. Then $\alpha(x) < t_1 < \alpha(0)$. Hence $x \in L(\alpha, t_1)$. This implies from the assumption that $L(\alpha, t_1)$ is an AB-ideal of X. Thus $0 \in L(\alpha, t_1)$, i.e., $\alpha(0) \le t_1$. This is a contradiction. Therefore, for each $x \in X$, $\alpha(0) \le \alpha(x)$.

Suppose further that there exist $x, y, z \in X$, $\alpha(x*z) > \max\{\alpha((x*y)*z), \alpha(y)\}$. Take $t_2 = \dfrac{\alpha(x*z) + \max\{\alpha((x*y)*z), \alpha(y)\}}{2} \in [0,1]$. Then $\max\{\alpha((x*y)*z), \alpha(y)\} < t_2 < \alpha(x*z)$. Hence $(x*y)*z, y \in L(\alpha, t_2)$ and $x*z \notin L(\alpha, t_2)$. This implies from the assumption that $L(\alpha, t_2)$ is an AB-ideal of X. Since $(x*y)*z, y \in L(\alpha, t_2)$ and $L(\alpha, t_2)$ is an AB-ideal of X, we have $x*z \in L(\alpha, t_2)$.

This is a contradiction. Therefore, $\alpha(x*z) \le \max\{\alpha((x*y)*z), \alpha(y)\}$, for all $x, y, z \in X$. □

Corollary 2 Let $(X, *, 0)$ be an AB-algebra and α a fuzzy set in X. If α is an anti-fuzzy AB-ideal of X, then for all $t \in \mathrm{Im}(\alpha)$, $L(\alpha, t)$ is an AB-ideal of X.

Corollary 3 Let I be an AB-ideal of an AB-algebra $(X, *, 0)$ and $k \in (0,1]$. Then there exists an anti-fuzzy AB-ideal f of X such that $L(f, t) = I$ for all $t < k$ and $L(f, t) = X$ for all $t \ge k$.

Proof Define a fuzzy set $f : X \to [0,1]$ by

$$f(x) = \begin{cases} 0, & x \in I, \\ k, & x \notin I. \end{cases}$$

Let $t \in [0,1]$. We will show that for each $t < k$, $L(f,t) = I$ and for each $t \geq k$, $L(f,t) = X$.

Case 1: $t < k$. Let $x \in L(f,t)$. Then $f(x) \leq t < k$. It follows from the definition of f that $f(x) = 0$ and $x \in I$. That is $L(f,t) \subseteq I$. Next, we let $x \in I$. Then $f(x) = 0 \leq t$. Thus $x \in L(f,t)$. Hence $I \subseteq L(f,t)$. Therefore, $L(f,t) = I$.

Case 2: $t \geq k$. Clearly, $L(f,t) \subseteq X$. Let $x \in X$. If $x \in I$, then $f(x) = 0 < t$. If $x \notin I$, then $f(x) = k \leq t$. This means that $x \in L(f,t)$. That is $X \subseteq L(f,t)$. Therefore, $L(f,t) = X$.

Moreover, since $L(f,t)$ is an AB-ideal of X for all $t \in [0,1]$, it can be concluded from Theorem 3 that f is an anti-fuzzy AB-ideal of X. □

Definition 8 The *complement* of a fuzzy set f in a nonempty set X, denoted by f^c, is defined to be $f^c = 1 - f$.

Theorem 4 Let α be a fuzzy set in an AB-algebra $(X, *, 0)$. Then α is an anti-fuzzy AB-ideal of X if and only if α^c is a fuzzy AB-ideal of X.

Proof Assume that α be an anti-fuzzy AB-ideal of X. Let $x, y, z \in X$. Since $\alpha(0) \leq \alpha(x)$, we have $1 - \alpha(0) \geq 1 - \alpha(x)$, i.e., $\alpha^c(0) \geq \alpha^c(x)$. Since $\alpha(x * z) \leq \max\{\alpha((x * y) * z), \alpha(y)\}$, it follows that

$$1 - \alpha(x * z) \geq 1 - \max\{\alpha((x * y) * z), \alpha(y)\}.$$

Hence $\alpha^c(x * z) \geq \min\{1 - \alpha((x * y) * z), 1 - \alpha(y)\} = \min\{\alpha^c((x * y) * z), \alpha^c(y)\}$. Therefore, α^c is a fuzzy AB-ideal. The proof of the converse part can be done by the similar argument. □

Definition 9 (Bejarasco *et al.,* 2019) Let $(X, \cdot, 0_X)$ and $(Y, *, 0_Y)$ be AB-algebras. A function $f : X \to Y$ is called an *AB-homomorphism* if $f(x \cdot y) = f(x) * f(y)$ for all $x, y \in X$. In addition, f is said to be an *AB-epimorphism* if f is surjective.

Lemma 1 (Bejarasco *et al.,* 2019) Let $(X, \cdot, 0_X)$ and $(Y, *, 0_Y)$ be AB-algebras and $f : X \to Y$ be an AB-homomorphism. Then $f(0_X) = 0_Y$ and for each $x, y \in X$, $x \leq y$ implies $f(x) \leq f(y)$.

Theorem 5 Let $(X, \cdot, 0_A)$ and $(Y, *, 0_B)$ be AB-algebras and let $f : X \to Y$ be an AB-epimorphism. Then for every anti-fuzzy AB-ideal β of Y, $\mu = \beta \circ f$ is an anti-fuzzy AB-ideal of X.

Proof Let $x, y, z \in X$. It follows from Lemma 1 that $f(0_X) \leq f(x)$. Since β is an anti-fuzzy AB-ideal, $\beta(f(0_X)) \leq \beta(f(x))$. Then $\mu(0_X) = \beta \circ f(0_X) = \beta(f(0_X)) \leq \beta(f(x)) = \mu(x)$. Moreover,

$$\begin{aligned}\mu(x\cdot z) &= \beta\circ f(x\cdot z)\\ &= \beta(f(x)*f(z))\\ &\le \max\{\beta((f(x)*f(y))*f(z)),\beta(f(y))\}\\ &= \max\{\beta\circ f((x\cdot y)\cdot z),\beta\circ f(y)\}\\ &= \max\{\mu((x\cdot y)\cdot z),\mu(y)\}.\end{aligned}$$

Hence μ is an anti-fuzzy AB-ideal of X. □

In the following definitions and theorems, we introduce the notion of Cartesian product of two anti-fuzzy ideals and prove some related properties.

Definition 10 Let $(A,\cdot,0_A)$ and $(B,*,0_B)$ be AB-algebras. Define a binary operation $\Diamond$ on $A\times B$ by

$$(x_1,x_2)\Diamond(y_1,y_2)=(x_1\cdot y_1,x_2*y_2)$$

for all $x_1,y_1\in A$ and $x_2,y_2\in B$.

It is straightforward to prove that $(A\times B,\Diamond,(0_A,0_B))$ is an AB-algebra.

Definition 11 For any anti-fuzzy ideals α and β of AB-algebras $(A,\cdot,0_A,)$ and $(B,*,0_B)$ respectively, we define the Cartesian product of anti-fuzzy AB-ideals of AB-algebras as follows:

$$(\alpha\times\beta)(x,y)=\max\{\alpha(x),\beta(y)\}$$

for all $(x,y)\in A\times B$.

Theorem 6 Let $(A,\cdot,0_A)$ and $(B,*,0_B)$ be any AB-algebras. If α and β are anti-fuzzy AB-ideals of AB-algebra A and B, respectively, then $\alpha\times\beta$ is an anti-fuzzy AB-ideal of $A\times B$.

Proof For any $(x_1,x_2)\in A\times B$,

$$(\alpha\times\beta)(0_A,0_B)=\max\{\alpha(0_A),\beta(0_B)\}\le\max\{\alpha(x_1),\beta(x_2)\}=(\alpha\times\beta)(x_1,x_2).$$

Next, let (x_1,x_2), (y_1,y_2) and $(z_1,z_2)\in A\times B$. Then

$$\begin{aligned}(\alpha\times\beta)((x_1,x_2)\Diamond(z_1,z_2)) &= (\alpha\times\beta)(x_1\cdot z_1,x_2*z_2)=\max\{\alpha(x_1\cdot z_1),\beta(x_2*z_2)\}\\ &\le \max\{\max\{\alpha((x_1\cdot y_1)\cdot z_1),\alpha(y_1)\},\max\{\beta((x_2*y_2)*z_2),\beta(y_2)\}\}\\ &= \max\{\max\{\alpha((x_1\cdot y_1)\cdot z_1),\beta((x_2*y_2)*z_2)\},\max\{\alpha(y_1),\beta(y_2)\}\}\\ &= \max\{(\alpha\times\beta)(((x_1,x_2)\Diamond(y_1,y_2))\Diamond(z_1,z_2)),(\alpha\times\beta)(y_1,y_2)\}.\end{aligned}$$

Hence $\alpha\times\beta$ is an anti-fuzzy AB-ideal of $A\times B$ which completes the proof. □

Discussion

We end this article with a short discussion that the converse of Theorem 6 is not true. Consider the following AB-algebra $(X, *, 0)$ as defined in the following table.

*	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	2	1	0	0
3	3	2	2	0

Let $k, l \in (0,1)$ such that $k \le l$. Define α and β as

$$\alpha(x) = \begin{cases} k & \text{if } x = 0 \\ l & \text{otherwise} \end{cases}$$

and

$$\beta(x) = \begin{cases} k & \text{if } x = 0 \\ 0 & \text{otherwise.} \end{cases}$$

By simple calculation, it can be concluded that $\alpha \times \beta$ is an anti-fuzzy AB-ideal of $X \times X$ but β is not an anti-fuzzy AB-ideal of X .

Moreover, we observe that if the Cartesian product of two fuzzy sets is an anti-fuzzy AB-ideal then either one of them must be an anti-fuzzy AB-ideal.

Lemma 2 Let $(A, \cdot, 0_A)$ and $(B, *, 0_B)$ be any AB-algebras. Let f and g be fuzzy sets in A and B , respectively. If $f \times g$ is an anti-fuzzy AB-ideal of $A \times B$, then the following statements hold.

1. Either $f(0_A) \le f(x)$ for all $x \in A$ or $g(0_B) \le g(y)$ for all $y \in B$.
2. If $f(0_A) \le f(x)$ for all $x \in A$ then $g(0_B) \le g(y)$ for all $y \in B$ or $g(0_B) \le f(x)$ for all $x \in A$.
3. If $g(0_B) \le g(y)$ for all $y \in B$ then $f(0_A) \le f(x)$ for all $x \in A$ or $f(0_A) \le g(y)$ for all $y \in B$.

Proof

1. To prove 1, we suppose on the contrary that $f(0_A) > f(x)$ for some $x \in A$ or $g(0_B) > g(y)$ for some $y \in B$. Then $(f \times g)(x, y) = \max\{f(x), g(y)\} < \max\{f(0_A), g(0_B)\} = (f \times g)(0_A, 0_B)$ which contradicts to the assumption that $f \times g$ is an anti-fuzzy AB-ideal of $A \times B$.

2. Assume that $f(0_A) \le f(x)$ for all $x \in A$. Again we prove this statement by using contradiction method. Suppose that $g(0_B) > g(y)$ for some $y \in B$ and $g(0_B) > f(x)$ for some $x \in A$. Then

$$g(0_B) > f(x) \ge f(0_A).$$

We have

$$\begin{aligned}(f \times g)(x, y) &= \max\{f(x), g(y)\} \\ &< \max\{g(0_B), g(0_B)\} \\ &= g(0_B) = \max\{f(0_A), g(0_B)\} \\ &= (f \times g)(0_A, 0_B)\text{, which is a contradiction.}\end{aligned}$$

3. The proof is similar to 2. □

Theorem 7 Let $(A, \cdot, 0_A,)$ and $(B, *, 0_B)$ be any AB-algebras. Let f and g be fuzzy sets in A and B, respectively. If $f \times g$ is an anti-fuzzy AB-ideal of $A \times B$, then either f is an anti-fuzzy AB-ideal of A or g is an anti-fuzzy AB-ideal of B.

Proof We prove this theorem by contradiction. Suppose that f is not an anti-fuzzy AB-ideal of A and g is not an anti-fuzzy AB-ideal of B. From 1. in Lemma 2, we have that either $f(0_A) \le f(x)$ for all $x \in A$ or $g(0_B) \le g(y)$ for all $y \in B$. Without loss of generality, we assume that $f(0_A) \le f(x)$ for all $x \in A$. By 2. in Lemma 2, we obtain that $g(0_B) \le g(y)$ for all $y \in B$ or $g(0_B) \le f(x)$ for all $x \in A$.

Case1: $g(0_B) \le f(x)$ for all $x \in A$.

Then for any $x \in A$, $(f \times g)(x, 0_B) = \max\{f(x), g(0_B)\} = f(x)$. It follows that

$$\begin{aligned}f(x \cdot z) &= (f \times g)(x \cdot z, 0_B) \\ &= (f \times g)(x \cdot z, 0_B * 0_B) \\ &= (f \times g)((x, 0_B) \Diamond (z, 0_B)) \\ &\le \max\{(f \times g)(((x, 0_B) \Diamond (y, 0_B)) \Diamond (z, 0_B)), (f \times g)(y, 0_B)\} \\ &= \max\{(f \times g)((x \cdot y) \cdot z, 0_B), (f \times g)(y, 0_B)\} \\ &= \max\{f((x \cdot y) \cdot z), f(y)\}.\end{aligned}$$

This implies that f is an anti-fuzzy AB-ideal of A, which contradicts to the assumption.

Case2: $g(0_B) \le g(y)$ for all $y \in B$.

Since f and g are not anti-fuzzy AB-ideal of A and B, respectively, there are $x, y, z \in A$ and $x', y', z' \in B$ such that

$$f(x\cdot z) > \max\left\{f\left(\left(x\cdot y\right)\cdot z\right), f(y)\right\} \text{ and } g(x^{'} * z^{'}) > \max\left\{g\left(\left(x^{'} * y^{'}\right) * z^{'}\right), g(y^{'})\right\}.$$

Thus

$$\max\left\{f(x\cdot z), g(x^{'} * z^{'})\right\} > \max\left\{\max\left\{f\left(\left(x\cdot y\right)\cdot z\right), f(y)\right\}, \max\left\{g\left(\left(x^{'} * y^{'}\right) * z^{'}\right), g(y^{'})\right\}\right\}.$$

However, since $f\times g$ is anti-fuzzy AB-ideal of $A\times B$, we have

$$\begin{aligned}
\max\left\{f(x\cdot z), g(x^{'} * z^{'})\right\} &= (f\times g)\left(x\cdot z, x^{'} * z^{'}\right)\\
&= (f\times g)\left((x, x^{'})\Diamond(z, z^{'})\right)\\
&\leq \max\left\{(f\times g)\left(\left((x, x^{'})\Diamond(y, y^{'})\right)\Diamond(z, z^{'})\right), (f\times g)(y, y^{'})\right\}\\
&= \max\left\{\max\left\{f\left((x\cdot y)\cdot z\right), g\left((x^{'} * y^{'}) * z^{'}\right)\right\}, \max\left\{f(y), g(y^{'})\right\}\right\}\\
&= \max\left\{\max\left\{f\left((x\cdot y)\cdot z\right), f(y)\right\}, \max\left\{g\left(\left(x^{'} * y^{'}\right) * z^{'}\right), g(y^{'})\right\}\right\}.
\end{aligned}$$

This is a contradiction.

From both cases, we can conclude that both f and g is an anti-fuzzy AB-ideals of A and B, respectively. □

Conclusions

We now have a lot of important results for anti-fuzzy AB-ideals of AB-algebras. For example, we obtain that anti-fuzzy AB-ideal is also an anti-fuzzy AB-subalgebra. In addition, we give characterizations of anti-fuzzy AB-ideals of AB-algebras by applying lower t-level sets. Moreover, we present that the complement of fuzzy AB-ideals are anti-fuzzy AB-ideals and vice versa. Finally, we show that the Cartesian product of two anti-fuzzy AB-ideals is an anti-fuzzy AB-ideal but the converse is not true.

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