

# สมบัติพื้นฐานบางประการของ $(m, n)$ -ไอดีลใน AG-กรุปพอยด์อันดับ

## Some Basic Properties of $(m, n)$ -Ideals in Ordered AG-Groupoids

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### บทคัดย่อ

ในบทความนี้เราศึกษา  $(0, 1)$ -ไอดีล,  $(0, 2)$ -ไอดีล และ  $(1, 1)$ -ไอดีลใน AG-กรุปพอยด์อันดับมีเอกลักษณ์ทางซ้าย ไอดีลข้างต้นเหล่านี้ได้ขยายมาจากไอดีลทางซ้ายและไอดีลทางขวาตามลำดับ ได้หาสมบัติเฉพาะบางประการของ  $(m, n)$ -ไอดีล,  $(0, 1)$ -ไอดีล,  $(0, 2)$ -ไอดีล และ  $(1, 1)$ -ไอดีลใน AG-กรุปพอยด์อันดับผลที่ได้จะขยายแนวคิดของ AG-กรุปพอยด์อันดับมีเอกลักษณ์ทางซ้าย

**คำสำคัญ** : AG-กรุปพอยด์ AG-กรุปพอยด์อันดับ  $(m, n)$ -ไอดีล  $(0, 1)$ -ไอดีล  $(0, 2)$ -ไอดีล

### Abstract

In this paper, we study  $(0, 1)$ ,  $(0, 2)$  and  $(1, 1)$ -ideals in ordered AG-groupoids with left identity. Those are extended from left and right ideals, respectively. Some characterizations of  $(m, n)$ ,  $(0, 1)$ ,  $(0, 2)$  and  $(1, 1)$ -ideals in ordered AG-groupoids. The obtained results extend the results on AG-groupoids with left identity.

**Keywords** : AG-groupoid, ordered AG-groupoid,  $(m, n)$ -ideal,  $(0, 1)$ -ideal,  $(0, 2)$ -ideal,  $(1, 1)$ -ideal

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## Introduction

A semigroup  $(S, \cdot)$  together with a partial order " $\leq$ " that is compatible with the semigroup operation, meaning that, for all  $a, b, c \in S$ , if  $a \leq b$ , then  $ac \leq bc$  and  $ca \leq cb$  is said to be an ordered semigroup (Birkhoff, 1967; Fuchs, 1963). A subsemigroup  $A$  of a semigroup  $S$  is called an  $(m, n)$ -ideal of  $S$  if  $A^m SA^n \subseteq A$ . This notion was first introduced and studied by Lajos (1961).

A groupoid  $S$  is called an Abel-Grassmann's groupoid, abbreviated as an AG-groupoid, if its elements satisfy the left invertive law (Holgate, 1992; Kazim and Naseeruddin, 1972), that is:  $(ab)c = (cb)a$ , for all  $a, b, c \in S$ . Several examples and interesting properties of AG-groupoids can be found in (Khan and Ahmad, 2010; Mushtaq, 1983; Mushtaq and Iqbal, 1990; Mushtaq and Iqbal, 1993). It has been shown in (Khan and Ahmad, 2010) that if an AG-groupoid contains a left identity then it is unique. It has been proved also that an AG-groupoid with right identity is a commutative monoid, that is, a semigroup with identity element. It is also known (Holgate, 1992) that in an AG-groupoid, the medial law, that is,

$$(ab)(cd) = (ac)(bd)$$

for all  $a, b, c, d \in S$  holds. Now we define the concepts that we will use. Let  $S$  be an AG-groupoid. By an AG-subgroupoid of (Mushtaq and Khan, 2009), we mean a non-empty subset  $A$  of  $S$  such that  $A^2 \subseteq A$ . A nonempty subset  $A$  of an AG-groupoid  $S$  is called a left (right) ideal of (Mushtaq and Khan, 2007) if  $SA \subseteq A$  ( $AS \subseteq A$ ). By two-sided ideal or simply ideal, we mean a non-empty subset of an AG-groupoid  $S$  which is both a left and a right ideal of  $S$ . The concept of an ordered AG-groupoid was first given by Khan and Faisal in (Khan and Faisal, 2001) which is in fact the generalization of an ordered semigroup. Furthermore, in this paper we characterize the  $(0,1)$ ,  $(0,2)$  and  $(1,1)$ -ideals in ordered AG-groupoids with left identity. Moreover, we investigate relationships  $(0,1)$  and  $(1,1)$ -ideals in ordered AG-groupoids with left identity.

## Preliminaries

In this section, we refer to (Shah, et al., 2010 and Shah, et al., 2011) for some elementary aspects and quote few definitions, and essential examples to step up this study. For more details, we refer to the papers in the references.

**Definition 2.1.** (Shah, et al., 2010 and Shah, et al., 2011) Let  $S$  be a nonempty set, " $\cdot$ " a binary operation on  $S$  and " $\leq$ " a relation on  $S$ .  $(S, \cdot, \leq)$  is called an ordered AG-groupoid if  $(S, \cdot)$  is an AG-groupoid,  $(S, \leq)$  is a partially ordered set and for all  $a, b, c \in S$ ,  $a \leq b$  implies that  $ac \leq bc$  and  $ca \leq cb$ .

**Lemma 2.2.**(Shah, et al.,2010 and Shah, et al.,2011) An ordered AG-groupoid  $S$  is an ordered semigroup if and only if  $a(bc) = (cb)a$ , for all  $a, b, c \in S$ .

**Proof.** See (Shah,et al.,2010 and Shah,et al.,2011).

Let  $(S, \cdot, \leq)$  be an ordered AG-groupoid. For  $A \subseteq S$ , let  $(A] = \{x \in S : x \leq a \text{ for some } a \in A\}$ . The following lemma are similar to the case of ordered AG-groupoids.

**Lemma 2.3.**(Shah,et al.,2010 and Shah,et al.,2011) Let  $S$  be an ordered AG-groupoid and let  $A, B$  be subsets of  $S$ . The following statements hold:

1. If  $A \subseteq B$ , then  $(A] \subseteq (B]$ .
2.  $(A](B] \subseteq (AB]$ .
3.  $((A](B]) \subseteq (AB]$ .

**Proof.** See (Shah,et al.,2010 and Shah,et al.,2011).

**Lemma 2.4.** Let  $S$  be an ordered AG-groupoid and let  $A, B$  be subsets of  $S$ . The following statements hold:

1.  $A \subseteq (A]$ .
2.  $((A]) = (A]$ .
3.  $((A](B]) = (AB]$ .

**Proof.** We leave the straightforward proof to the reader.

**Definition 2.5.**(Shah,et al.,2010 and Shah,et al.,2011) A nonempty subset  $A$  of an ordered AG-groupoid  $S$  is called a AG-subgroupoid of  $S$  if  $AA \subseteq A$ .

**Definition 2.6.**(Shah,et al.,2010 and Shah,et al.,2011) A nonempty subset  $A$  of an ordered AG-groupoid  $S$  is called a left ideal of  $S$  if  $(A] \subseteq A$  and  $SA \subseteq A$  and called a right ideal of  $S$  if  $(A] \subseteq A$  and  $AS \subseteq A$ . A nonempty subset  $A$  of  $S$  is called an ideal of  $S$  if  $A$  is both left and right ideal of  $S$ .

**Lemma 2.7.**(Shah,et al.,2010 and Shah,et al.,2011) Let  $S$  be an ordered AG-groupoid with left identity. Then every right ideal of  $S$  is a left ideal of  $S$ .

**Proof.** See (Shah,et al.,2010 and Shah,et al.,2011).

**Lemma 2.8.**(Shah,*et al.*,2010 and Shah,*et al.*,2011) Let  $S$  be an ordered AG-groupoid with left identity and  $A \subseteq S$ . Then  $S(SA) = SA$  and  $S(SA] \subseteq (SA]$ .

**Proof.** See (Shah, *et al.*,2010 and Shah,*et al.*,2011).

**Lemma 2.9.**(Shah,*et al.*,2010 and Shah,*et al.*,2011) Let  $S$  be an ordered AG-groupoid with left identity and  $a \in S$ . Then  $\langle a \rangle = (Sa]$ .

**Proof.** See (Shah, *et al.*,2010 and Shah,*et al.*,2011).

### $(m, n)$ -Ideals

A nonempty subset  $A$  of an ordered AG-groupoid  $(S, \cdot, \leq)$  is called an AG-subgroupoid of  $S$  if  $AA \subseteq A$ , that is,  $xy \in A$  for all  $x, y \in A$ . If  $A$  is any nonempty subset of an ordered AG-groupoid  $S$ , then we define  $A^n = (\dots ((AA)A)\dots)A$ , where  $AA$  is a usual product and  $n$  is any positive integer.

**Definition 3.1.** (Amjid, *et al.*, 2017) Let  $(S, \cdot, \leq)$  be an ordered AG-groupoid and let  $m, n$  be nonnegative integers. An AG-subgroupoid  $A$  of  $S$  is called an  $(m, n)$ -ideal of  $S$  if the following hold:

1.  $(A^m S)A^n \subseteq A$ ;
2.  $(A] = A$ , that is, for  $x \in A$  and  $y \in S$ ,  $y \leq x$  implies  $y \in A$ .

From Definition 3.1, if  $m = 1, n = 1$ , then  $A$  is called a bi-ideal of  $S$ .

**Remark.** It is clear that if  $A$  is an ideal of  $S$ , then  $A$  is an  $(m, n)$ -ideal of  $S$ .

**Theorem 3.2.** If  $A^2$  is an AG-subgroupoid of an ordered AG-groupoid with left identity  $S$ , then  $(A^2 \cup (A^2 S)A^2]$  is an  $(1,1)$ -ideal of  $S$ .

**Proof.** Assume that  $(S, \cdot, \leq)$  is an ordered AG-groupoid with left identity. Let  $A^2$  be an AG-subgroupoid of  $S$ . By Lemma 2.4, we have  $(A^2 \cup (A^2 S)A^2] = ((A^2 \cup (A^2 S)A^2])$ . Then

$$\begin{aligned}
 (A^2 \cup (A^2 S)A^2](A^2 \cup (A^2 S)A^2] &\subseteq (A^2(A^2 \cup (A^2 S)A^2) \cup ((A^2 S)A^2)(A^2 \cup (A^2 S)A^2)] \\
 &= (A^2 A^2 \cup A^2((A^2 S)A^2) \cup ((A^2 S)A^2)A^2 \cup \\
 &\quad ((A^2 S)A^2)((A^2 S)A^2)] \\
 &\subseteq (A^2 \cup (A^2 S)(A^2 A^2) \cup (A^2 A^2)(A^2 S) \cup \\
 &\quad (A^2(A^2 S))(A^2(A^2 S))]
 \end{aligned}$$

$$\begin{aligned}
 &\subseteq ((A^2 \cup (A^2S)A^2 \cup (A^2A^2)(A^2S) \cup \\
 &\quad (A^2((A^2(A^2S))S))A^2] \\
 &\subseteq (A^2 \cup (A^2S)A^2 \cup ((A^2S)A^2)A^2 \cup (A^2((A^2(A^2S))S))A^2] \\
 &\subseteq (A^2 \cup (A^2S)A^2 \cup A^2 \cup (A^2S)A^2 \cup A^2 \cup (A^2S)A^2] \\
 &= (A^2 \cup (A^2S)A^2]
 \end{aligned}$$

and

$$\begin{aligned}
 ((A^2 \cup (A^2S)A^2]S)(A^2 \cup (A^2S)A^2] &= ((A^2 \cup (A^2S)A^2](S))(A^2 \cup (A^2S)A^2] \\
 &= ((A^2S \cup ((A^2S)A^2)S)(A^2 \cup (A^2S)A^2] \\
 &= ((A^2S)(A^2 \cup (A^2S)A^2) \cup (((A^2S)A^2)S)(A^2 \cup (A^2S)A^2)] \\
 &= ((A^2S)A^2 \cup (A^2S)((A^2S)A^2) \cup (((A^2S)A^2)S)A^2 \cup \\
 &\quad (((A^2S)A^2)S)((A^2S)A^2)] \\
 &= ((A^2S)A^2 \cup (A^2(A^2S))(SA^2) \cup ((SA^2)(A^2S))A^2 \cup \\
 &\quad ((SA^2)(A^2S))((A^2S)A^2)] \\
 &= ((A^2S)A^2 \cup ((SA^2)(A^2S))A^2 \cup (A^2((SA^2)S))A^2 \cup \\
 &\quad (A^2((SA^2)S))((A^2S)A^2)] \\
 &= ((A^2S)A^2 \cup (A^2((SA^2)S))A^2 \cup (A^2((SA^2)S))A^2 \cup \\
 &\quad (((A^2S)A^2)((SA^2)S))A^2] \\
 &= ((A^2S)A^2 \cup (A^2((SA^2)S))A^2 \cup (A^2((SA^2)S))A^2 \cup \\
 &\quad (((S(SA^2))(A^2(A^2S)))A^2] \\
 &= ((A^2S)A^2 \cup (A^2((SA^2)S))A^2 \cup (A^2((SA^2)S))A^2 \cup \\
 &\quad (A^2(((S(SA^2))(A^2S)))A^2] \\
 &\subseteq ((A^2S)A^2 \cup (A^2S)A^2 \cup (A^2S)A^2 \cup (A^2S)A^2] \\
 &= ((A^2S)A^2] \\
 &\subseteq (A^2 \cup (A^2S)A^2].
 \end{aligned}$$

Therefore  $(A^2 \cup (A^2S)A^2]$  is an  $(1,1)$ -ideal of  $S$ .

**Theorem 3.3.** If  $(S, \cdot, \leq)$  is an ordered AG-groupoid with left identity and  $\emptyset \neq A \subseteq S$ , then  $(A^2 \cup SA^2]$  is an ideal of  $S$ .

**Proof.** Assume that  $(S, \cdot, \leq)$  is an ordered AG-groupoid with left identity. Let  $\emptyset \neq A \subseteq S$ . Lemma 2.4, we have  $(A^2 \cup SA^2] = ((A^2 \cup SA^2])$ . Then

$$S(A^2 \cup SA^2] = (S](A^2 \cup SA^2]$$

$$\begin{aligned}
 &\subseteq (SA^2 \cup S(S^2A^2)] \\
 &= (SA^2 \cup S(A^2S^2)] \\
 &= (SA^2 \cup A^2(SS^2)] \\
 &= (SA^2 \cup A^2S^2] \\
 &= (SA^2 \cup S^2A^2] \\
 &\subseteq (A^2 \cup SA^2 \cup A^2 \cup SA^2] \\
 &= (A^2 \cup SA^2]
 \end{aligned}$$

and

$$\begin{aligned}
 (A^2 \cup SA^2]S &= (A^2 \cup SA^2](S] \\
 &\subseteq (A^2S \cup (SA^2)S] \\
 &= (A^2S \cup (S^2A^2)S] \\
 &= (A^2S^2 \cup (S^2A^2)S] \\
 &= (S^2A^2 \cup (A^2S^2)S] \\
 &= (SA^2 \cup (SS^2)A^2] \\
 &\subseteq (A^2 \cup SA^2 \cup A^2 \cup SA^2].
 \end{aligned}$$

Therefore  $(A^2 \cup SA^2]$  is an ideal of  $S$ .

**Theorem 3.4.** Let  $(S, \cdot, \leq)$  be an ordered AG-groupoid with left identity and  $\emptyset \neq A \subseteq S$ . If  $A^2$  is a  $(0, 2)$ -ideal of  $S$ , then  $A^2$  is an ideal of some ideal of  $S$ .

**Proof.** Let  $(S, \cdot, \leq)$  be an ordered AG-groupoid with left identity and  $\emptyset \neq A \subseteq S$ . If  $A^2$  is a  $(0, 2)$ -ideal of  $S$ , then

$$\begin{aligned}
 (A^2 \cup SA^2]A^2 &\subseteq (A^2 \cup SA^2](A^2] \\
 &\subseteq (A^2A^2 \cup (SA^2)A^2] \\
 &\subseteq (SA^2 \cup A^2A^2] \\
 &\subseteq (A^2 \cup A^2] \\
 &\subseteq (A^2] \\
 &= A^2
 \end{aligned}$$

and

$$\begin{aligned}
 A^2(A^2 \cup SA^2] &\subseteq (A^2](A^2 \cup SA^2] \\
 &\subseteq (A^2A^2 \cup A^2(SA^2)] \\
 &\subseteq (SA^2 \cup A^2(SA^2)]
 \end{aligned}$$

$$\begin{aligned}
&\subseteq (A^2 \cup A^2 A^2] \\
&\subseteq (A^2] \\
&= A^2.
\end{aligned}$$

Since  $(A^2 \cup SA^2]$  is an ideal of  $S$ , we get  $A^2$  is an ideal of  $(A^2 \cup SA^2]$ .

**Theorem 3.5.** Let  $(S, \cdot, \leq)$  be an ordered AG-groupoid with left identity and  $\emptyset \neq A \subseteq S$ . If  $A$  is a left ideal of some left ideal of  $S$ , then  $A$  is an  $(0, 2)$ -ideal of  $S$ .

**Proof.** Assume that  $A$  is a left ideal of a left ideal  $L$  of  $S$ . Then

$$\begin{aligned}
SA^2 &\subseteq S^2(LA) \\
&= (AL)S^2 \\
&= (SL)A \\
&\subseteq LA \\
&\subseteq A.
\end{aligned}$$

Let  $x \in A$  and  $y \in S$  such that  $y \leq x$ . Since  $x \in L$ , we have  $y \in L$ . The assumption implies  $y \in A$ . Hence  $A$  is an  $(0, 2)$ -ideal of  $S$ .

**Theorem 3.6.** Let  $(S, \cdot, \leq)$  be an ordered AG-groupoid with left identity and  $\emptyset \neq A \subseteq S$ . If  $A$  is an  $(1, 2)$ -ideal of  $S$ , then  $A$  is a left ideal of some  $(1, 1)$ -ideal of  $S$ .

**Proof.** Assume that  $A$  is an  $(1, 2)$ -ideal of  $S$ . Then

$$\begin{aligned}
(A^2 \cup (A^2 S)A^2]A &\subseteq (A^2 \cup (A^2 S)A^2](A] \\
&= (A^2 A \cup ((A^2 S)A^2)A] \\
&\subseteq (A^2(SA) \cup (A^2(SA^2))A] \\
&= ((AS)A^2 \cup (A(SA^2))A^2] \\
&\subseteq (A \cup A] \\
&= (A] \\
&= A.
\end{aligned}$$

Hence  $A$  is a left ideal of  $(A^2 \cup (A^2 S)A^2]$ .

**Theorem 3.7.** Let  $(S, \cdot, \leq)$  be an ordered AG-groupoid with left identity and  $\emptyset \neq A \subseteq S$ . If  $A$  is a left ideal of some  $(1, 1)$ -ideal of  $S$ , then  $A$  is an  $(1, 1)$ -ideal of some ideal of  $S$ .

**Proof.** Let  $A$  be a left ideal of an  $(1, 1)$ -ideal  $B$  of  $S$ . Then

$$(A(A^2 \cup SA^2])A \subseteq (AA^2 \cup A(SA^2))A$$

$$\begin{aligned}
 &= ((AA^2)A \cup (A(SA^2))A] \\
 &= (A \cup (S(AA^2))A] \\
 &= (A \cup ((A^2A)S)A] \\
 &= (A \cup ((A^2A)S)A] \\
 &= (A \cup (S(AA^2))A] \\
 &= (A \cup (A(SA^2))A] \\
 &\subseteq (A \cup (B(SB))A] \\
 &\subseteq (A \cup BA] \\
 &\subseteq (A \cup A] \\
 &= (A] \\
 &= A.
 \end{aligned}$$

Hence  $A$  is a left ideal of  $(A^2 \cup (A^2S)A^2]$ .

**Lemma 3.8.** If  $(S, \cdot, \leq)$  is an ordered AG-groupoid with left identity and  $\emptyset \neq A \subseteq S$ , then  $(A^2 \cup SA^2]$  is a right ideal of  $S$ .

**Proof.** Assume that  $(S, \cdot, \leq)$  is an ordered AG-groupoid with left identity. By Lemma 2.4, we have  $(A^2 \cup A^2S] = ((A^2 \cup A^2S])$ . Then

$$\begin{aligned}
 (A^2 \cup A^2S]S &= (A^2 \cup A^2S](S) \\
 &\subseteq (A^2S \cup (A^2S)S] \\
 &= (A^2S \cup SA^2] \\
 &= (A^2S \cup A^2S] \\
 &= (A^2S] \\
 &\subseteq (A^2 \cup A^2S].
 \end{aligned}$$

Therefore  $(A^2 \cup SA^2]$  is a right ideal of  $S$ .

**Theorem 3.9.** Let  $(S, \cdot, \leq)$  be an ordered AG-groupoid with left identity and  $\emptyset \neq A \subseteq S$ . If  $A$  is an  $(1,1)$ -ideal of some ideal of  $S$ , then  $A$  is an  $(0,1)$ -ideal of some right ideal of  $S$ .

**Proof.** Assume that  $A$  is an  $(1,1)$ -ideal of an ideal  $L$  of  $S$ . Note that  $(A^2 \cup A^2S]$  is a right ideal of  $S$ . Consider

$$\begin{aligned}
 (A^2 \cup A^2S]A &\subseteq (A^2 \cup A^2S](A) \\
 &= (A^2A \cup (A^2S)A] \\
 &\subseteq ((AL)A \cup (SA^2)A] \\
 &\subseteq (A \cup (A(SA))A]
 \end{aligned}$$



$$\begin{aligned}
&\subseteq (A \cup (A(SL))A] \\
&\subseteq (A \cup (AL)A] \\
&\subseteq (A \cup A] \\
&= (A] \\
&= A.
\end{aligned}$$

Hence  $A$  is an  $(0,1)$ -ideal of  $(A^2 \cup A^2S]$ .

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