แผนภูมิควบคุมสำหรับกระบวนการผลิตที่มีการแจกแจงแบบปัวซงวางนัยทั่วไป ซึ่งมีศูนย์มากกับการกระจายที่มากเกินจริง

Control Charts for Zero-Inflated Generalized Poisson Process with Over-dispersion

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งานวิจัยนี้มีจุดประสงค์เพื่อศึกษาแผนภูมิควบคุมสำหรับตรวจจับการเปลี่ยนแปลงค่าเฉลี่ยจำนวนรอยตำหนิ (λ) เมื่อกระบวนการผลิตมีการแจกแจงแบบปัวซงวางนัยทั่วไปที่มีศูนย์มาก (*ZIGP*) ซึ่งมีการกระจายที่มากเกินจริง แผนภูมิแรกคือแผนภูมิควบคุมผลรวมสะสมเมื่อค่าสถิติผลรวมสะสมสร้างบนพื้นฐานอัตราส่วนล็อกภาวะน่าจะเป็นเรียก λ_z - *CUSUM chart* แผนภูมิที่สองคือแผนภูมิควบคุมค่าเฉลี่ยเคลื่อนที่ถ่วงน้ำหนักเอ็กซ์โปเนนเชียลสร้างบนพื้นฐาน *ZIGP* เรียก $EWMA_Z$*-chart* แผนภูมิที่สามคือแผนภูมิควบคุมจำนวนรอยตำหนิต่อหน่วยสร้างบนพื้นฐาน *ZIGP* เรียก c_Z*-chart* ประสิทธิภาพของแผนภูมิควบคุมพิจารณาที่ค่าความยาววิ่งเฉลี่ย ผลการวิจัยเมื่อมีการเปลี่ยนแปลงในค่า λ พบว่า $EWMA_Z$*-chart* มีประสิทธิภาพในทุกระดับค่าของ λ สัดส่วนของรอยตำหนิเป็นศูนย์ (ω) การเปลี่ยนแปลงค่าเฉลี่ย (ρ) และการกระจายที่มากเกินจริง (φ)

คำสำคัญ : การกระจายที่มากเกินจริง, การแจกแจงแบบปัวซงวางนัยทั่วไปที่มีศูนย์มาก, แผนภูมิควบคุมค่าเฉลี่ยเคลื่อนที่ถ่วงน้ำหนักเอ็กซ์โปเนนเชียล

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Abstract

This paper aims to study charts for detecting the mean of nonconformities (λ) shifts based on the zero-inflated generalize poisson (*ZIGP*) process with over-dispersion. The first chart is the same as a *CUSUM-chart* where *CUSUM* statistics constructed base on log-likelihood ratio called, λ_z - *CUSUM chart*, the second chart is the same as a *EWMA-chart* based on *ZIGP* process called, $EWMA_Z$*-chart*, the third chart is the same as a *c-chart* based on *ZIGP* process called, c_Z*-chart*. The performance of control the charts was considered from the average run length. The research result shows that for the λ shifts process, the $EWMA_Z$*-chart* is the best for all level of λ, proportion zero (ω), mean shift (ρ) and over-dispersion (φ).

Keywords: over-dispersion, zero-inflated generalized poisson distribution, The exponentially weighted moving average control chart

Introduction

Traditional poisson distribution is inappropriate for an excess number of zero nonconformities in processes. This paper focuses on zero-inflated generalize poisson (*ZIGP*) distribution. The *ZIGP* developed from generalized poisson distribution (*GPD*) has two parameters λ and φ, where λ is the mean of the nonconformities in a sample unit and φ is the over-dispersion (Famoye and Singh, 2003). *ZIGP* distribution is the generalized poisson distribution combined with a mixture of the proportion of zero nonconformities (ω) (Famoye and Singh, 2006).

The *c-chart,* base on the poisson distribution is used to monitor nonconformities in processes. However, if there is an excess number of zero nonconformities in the processes then the *c-chart* is an unsuitable control chart. This paper is interested in the cumulative sum chart (*CUSUM-chart*) and the exponentially weighted moving average (*EWMA-chart*). Because the *CUSUM-chart* and the *EWMA-chart* are efficient charts to monitor smaller shifts in the process, Montgomery, 2005. Page, 1954 first proposed the *CUSUM-chart* and other authors, Gan, 1990 and Lucas, 1976. Katemee and Mayureesawan, 2013 constructed the *CUSUM-chart* for the *ZIGP* process. They show, that the ω - *CUSUM chart*, λ - *CUSUM chart* and φ - *CUSUM chart*, are performed for detecting the mean shift of individual parameter. Gan, 1991 studied the *EWMA-chart* for detecting the λ of the Poisson process. He founded that the *EWMA-chart* was the best for detecting mean shift in λ.

The aim of this paper is to study the influence of *CUSUM-chart*, *EWMA-chart* and *c-chart* based on the *ZIGP* process, in the case where the variance is greater than the mean, called over-dispersion. All of these three charts are used to detect changes in individual parameters of the mean of nonconformities (λ). The measure of control chart performance was considered from the average run length (*ARL*). The *CUSUM-chart* and the *EWMA-chart* based on the *ZIGP* process were compared with the *c-chart* based on the *ZIGP* distribution.

Methods

The Zero-Inflated Generalized Poisson (*ZIGP*) distribution

The probability function is given by: (Famoye and Singh, 2006)

$$P(Y=y)=\begin{cases}\omega+(1-\omega)\exp(-\lambda\varphi), & y=0\\ (1-\omega)\exp\left(-\frac{1}{\varphi}(\lambda+y(\varphi-1))\right)\frac{\lambda\left(\lambda+y(\varphi-1)\right)^{y-1}}{\varphi^{y}y!}, & y>0\end{cases} \quad (1)$$

where y = the random variables of nonconformities in a sample unit,

λ = the mean of nonconformities in a sample unit based on the *ZIGP* distribution,

ω = a measure of the extra proportion of zero nonconformity in a sample unit,

φ = the over-dispersion for *ZIGP* distribution, and

$$E(Y)=(1-\omega)\lambda \text{ and } V(Y)=(1-\omega)\lambda\left(\varphi^{2}+\lambda\omega\right). \quad (2)$$

The Cumulative Sum Chart based on a *ZIGP* distribution (λ_z - *CUSUM chart*)

1. The λ_z - *CUSUM chart* is a *CUSUM chart* for detecting shifts in a parameter λ . The cumulative sum statistics constructed base on log-likelihood ratio for plotting on the λ_z - *CUSUM chart* $\left(A_i\right)$ defined as: (Katemee and Mayureesawan, 2013)

$$A_i=\max\left(0,A_{i-1}+D_i\right), \quad i=1,2,\ldots \quad (3)$$

The head start value of the cumulative sum statistics $\left(A_0\right)=0$ and D_i is the log-likelihood ratio of *ZIGP* distribution for a shift in parameter λ (λ_1) defined as follows:

$$D_i = D(y_i) = \begin{cases} \ln \dfrac{\omega_0 + (1-\omega_0)\, e^{(-\lambda_1 \varphi_0)}}{\omega_0 + (1-\omega_0)\, e^{(-\lambda_0 \varphi_0)}}, & y_i = 0 \\ \dfrac{\lambda_0 - \lambda_1}{\varphi_0} + \ln(\dfrac{\lambda_1}{\lambda_0}) + (y_i - 1)\ln \dfrac{(\lambda_1 + y_i(\varphi_0 - 1))}{(\lambda_0 + y_i(\varphi_0 - 1))}, & y_i > 0 \end{cases} \quad (4)$$

where y_i = the observations of y taken at the time *i*,

λ_0 = the in-control value of the mean number of nonconformities for *ZIGP* distribution,

λ_1 = the out-of-control values of the mean number of nonconformities for *ZIGP* distribution,

ω_0 = the in-control value of the proportion of zero nonconformity for *ZIGP* distribution,

φ_0 = the in-control value of the over dispersion for *ZIGP* distribution,

The λ_Z - *CUSUM chart* will signals in the process when $A_i > H_\lambda$, where H_λ is the *UCL* of the λ_Z - *CUSUM chart* i that is determined based on require in-control performance.

The Exponentially Weighted Moving Average chart is based on *ZIGP* distribution (*EWMA$_Z$-chart*)

The *EWMA$_Z$-chart* is the same as a *EWMA-chart* based on detection of changes in parameter λ of the *ZIGP* distribution. The *EWMA* statistics for plotting on the *EWMA$_Z$-chart* (Z_i) are defined as

$$Z_i = \xi y_i + (1-\xi) Z_{i-1}, \quad i = 1,2,\ldots \quad (5)$$

The head start value of the *EWMA* statistics $(Z_0) = \lambda_0$

where ξ = a constant that determinations must satisfy $0 < \xi \leq 1$,

y_i = the observation of y taken at time i,

λ_0 = the in-control value of the mean number of nonconformities for *ZIGP* distribution,

The *EWMA$_Z$-chart* will signal in the process when $Z_i > H_{EWMA}$, where H_{EWMA} is the *UCL* of the *EWMA$_Z$-chart* that is determined based on required in-control performance.

The Shewhart control chart of nonconformities is based on *ZIGP* distribution (*c$_Z$-chart*)

The *c$_Z$-chart* is the same as a *c-chart* based on detecting shifts in parameters of the *ZIGP* distribution. The upper control limit (*UCL*) of the *c$_Z$-chart* is $c + L\sqrt{c}$, where *c* is assumed to be the mean number of nonconformities if the mean of the probability distribution is known and L is the coefficient of control limit of *c-chart*. The c_Z*-chart* will signals when any observations of nonconformities (y_i) is greater than H_c, where H_c is the *UCL* of the c_Z*-chart* that is selected matching the desired in-control performance.

Simulation Results

In the simulations where the average run length (*ARL*) was close to the target, the mean number of nonconformities of: (λ_0) = 2.0 and 4.0 was set. The proportions of zero nonconformity were: (ω_0) = 0.3 and 0.5. The over dispersions were: (φ_0) = 1.2 and 1.4. The out-of-control values of the mean number of nonconformities were:$\lambda_1 = \lambda_0 + \rho$, where the mean shifts were: (ρ) = 2, 3, 4, 5, 6, 7 and 8. The *ARL* was the criteria for evaluating the performance of the control charts. The seven steps of the research process were:

1. The R program was used to simulate the number of nonconforming items for a *ZIGP* where the parameters were$\left(n,\lambda_0,\varphi_0,\omega_0\right)$.

2. The three value of upper control limit were: the value of H_λ for λ_z- *CUSUM chart*, the value of H_{EWMA} for $EWMA_Z$*-chart* and value of H_c for c_Z*-chart*. Each value of λ_0,ω_0 and φ_0 was set in the study. However, the values of H_λ, H_{EWMA} and H_c were changed to match ARL_0 = 370 for all charts. Calculations, based on 100,000 replications, for the average upper control limit that were in each the parameters.

3. For the λ_z- *CUSUM chart* calculations of the log-likelihood ratios for the cumulative sum statistics (D_i) were plotted in the A_i value from (3). The $EWMA_Z$*-chart*, calculated the Z_i value from (5) were used for the $EWMA_Z$*-chart*. The c_Z*-chart*, simulation of the numbers of nonconforming (y_i) were used for the c_Z*-chart*.

4. The λ_z- *CUSUM char*, investigated the A_i value with H_λ to find the run length (*RL*). The $EWMA_Z$*-chart*, investigated the Z_i value with H_{EWMA} value. The c_Z*-chart*, investigated the y_i value with H_c value. Consider investigating the A_i, Z_i and y_i that are out-of-control points. When there were points outside the control limit, then they were stored in the observations before a point indicated an out-of-control for the run length (*RL*) calculation. If they were at *i* statistics indicated an out-of-control then *RL* = *i* -1.

5. One hundred thousand (100,000) replications for the average run length (*ARL*) were computed form Steps 3 to 4 for each of the charts.

6. When contrasted the performance of control charts that gave a low *ARL*, then the control charts were determined efficient.

7. Changing during a parameters value in the study to completely.

Results and Dicussion

An overview of the control charts shows they were proficient for all levels of the parameters and all levels of shifts. Table 1 defines the levels for parameter shifts when λ_0 = 2, ω_0 = 0.3 and φ_0 = 1.2.

Table 2 shows the upper control limit of λ_z - *CUSUM chart* (H_λ), *EWMA*$_Z$*-chart* (H_{EWMA}) and c_Z*-chart* (H_c) matched with the ARL_0 = 370. It can be seen that all values of λ_0 and φ_0. If H_λ increased then ω_0 was decreasing. However, H_{EWMA} and ω_0 were going in the same direction. The results showed that the c_Z*-chart* returned unsuitable values of ARL_0, therefore this paper is not shown.

Table 3 and Fig. 1 show the ARL_1 of λ_z - *CUSUM chart* and *EWMA*$_Z$*-chart* for shift in parameter λ. The results found that for all λ_0, φ_0, ω_0 and level of a shift, the *EWMA*$_Z$*-chart* returned low values of ARL_1. That the *EWMA*$_Z$*-chart* gave a better performance than λ_z - *CUSUM chart* because it was able to detect the shift faster.

Table 1 Defines the levels for parameter shifts in λ when λ_0 = 2

Levels of Shifts	1	2	3	4	5	6	7
parameter shifts in λ $(\lambda_1 = \lambda_0 + \rho)$	$\lambda_1 = 4$	$\lambda_1 = 5$	$\lambda_1 = 6$	$\lambda_1 = 7$	$\lambda_1 = 8$	$\lambda_1 = 9$	$\lambda_1 = 10$

Table 2 The upper control limit H_λ, H_{EWMA} and H_c were matching with the desired in-control performance for all levels of the λ_0, ω_0 and φ_0

λ_0	2	2	2	2	4	4	4	4
ω_0	0.30	0.50	0.30	0.50	0.30	0.50	0.30	0.50
φ_0	1.2	1.2	1.4	1.4	1.2	1.2	1.4	1.4
H_λ	3.50	3.20	4.68	4.49	3.00	2.70	3.31	2.90
ARL_0	370.3	371.5	369.7	366.9	369.2	369.2	372.8	372.7
H_c	8	8	10	9	12	11	14	14
ARL_0	386.2	482.1	489.6	316.2	397.4	290.8	338.6	439.7
H_{EWMA}	5.155	5.5	6.107	6.573	8.038	8.252	9.15	9.48
ARL_0	370.1	370.4	370.1	370.8	370.7	370.0	370.7	370.3

Table 3 The ARL_1 of λ_z - *CUSUM chart* and $EWMA_Z$*-chart* for shift in parameter λ

λ_0	ρ	$\varphi_0 = 1.2$				$\varphi_0 = 1.4$			
		$\omega_0 = 0.3$		$\omega_0 = 0.5$		$\omega_0 = 0.3$		$\omega_0 = 0.5$	
		λ_z - *CUSUM*	$EWMA_Z$ $\xi = 0.5$	λ_z - *CUSUM*	$EWMA_Z$ $\xi = 0.613$	λ_z - *CUSUM*	$EWMA_Z$ $\xi = 0.5$	λ_z - *CUSUM*	$EWMA_Z$ $\xi = 0.613$
2	2	31.647	16.740	38.757	24.131	47.147	27.834	61.272	39.654
	3	12.774	7.487	17.121	11.063	18.651	12.906	22.751	19.089
	4	7.697	4.315	10.449	6.301	10.607	7.153	13.649	10.884
	5	5.337	2.837	7.635	4.135	7.826	4.640	9.678	7.124
	6	4.113	2.034	6.042	3.010	5.965	3.220	7.571	4.932
	7	3.828	1.504	4.980	2.326	4.805	2.418	6.547	3.678
	8	3.189	1.204	4.649	1.850	4.408	1.899	5.738	2.915
λ_0	ρ	λ_z - *CUSUM*	$EWMA_Z$ $\xi = 0.5$	λ_z - *CUSUM*	$EWMA_Z$ $\xi = 0.6$	λ_z - *CUSUM*	$EWMA_Z$ $\xi = 0.5$	λ_z - *CUSUM*	$EWMA_Z$ $\xi = 0.6$
4	2	43.49	30.056	53.792	41.106	57.515	45.086	98.951	60.790
	3	18.350	13.637	23.216	19.747	23.887	21.784	31.581	31.436
	4	10.088	7.757	13.662	11.069	13.908	12.270	18.272	18.148
	5	7.574	4.891	9.962	7.235	10.037	7.725	13.342	11.493
	6	5.670	3.459	7.784	5.170	7.827	5.305	10.112	7.998
	7	4.803	2.606	6.803	3.829	6.231	3.899	8.338	5.959
	8	4.159	1.988	5.938	2.955	5.563	3.040	7.638	4.591

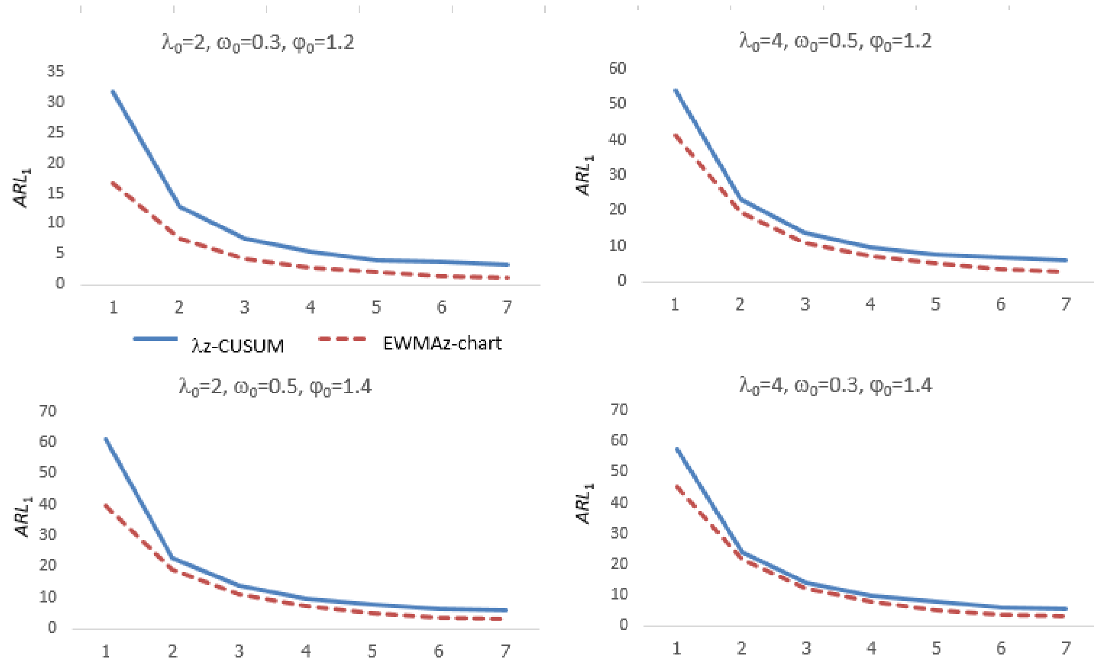

Figure 1 The ARL_1 of the, λ_z - *CUSUM chart* and *$EWMA_Z$-chart* for the shifts in parameter λ

Conclusions

The purpose of this paper was to study of the *CUSUM chart*, *EWMA-chart* and *c-chart* to detect parameter shifts of λ for the *ZIGP* process with over-dispersion. The three charts were, the λ_z - *CUSUM chart*, *$EWMA_Z$-chart* and *c_Z-chart*. The λ_z - *CUSUM chart* was a *CUSUM chart* where cumulative sum statistics constructed base on the log-likelihood ratio. The *$EWMA_Z$-chart* was a *EWMA-chart* based on the *ZIGP* process. The *c_Z-chart* was a *c-chart* based on the *ZIGP* process. The average run length (*ARL*) of these chats were considered. The results of the comparisons are summarized as follows, the *$EWMA_Z$-chart* performs better than the λ_z - *CUSUM chart* because its performance in detection in mean shift is faster for all level values of the parameters in the processes.

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References

Famoye, F. and Singh, K. P. (2003). *On inflated generalized Poisson regression models*. Advance and Applied Statistics, *3*, 145–158.

Famoye, F. and Singh, K. P. (2006). *Zero-Inflated Generalized Poisson Regression Model with an Application to Domestic Violence Data*. Journal of Data Science, *4*, 117-130.

Gan, F.F. (1990). *Monitoring Poisson Observations Using Modified Exponentially Weighted Moving Average Control Charts*. Communications in Statistics-Simulation and Computation, *19*, 103-104.

Gan, F.F. (1991). *An Optimal Design of CUSUM Quality Control Charts*. Journal of Quality Control, *23*, 279-286.

Katemee, N. and Mayureesawan, T. (2013). *CUSUM Charts for Zero-Inflated Generalized Poisson Process*. Far East Journal of Mathematical Sciences, *77*, 225-243.

Lucus, J.M. (1976). *The Design and Use of V-mask Control Schemes*. Journal of Quality Control*, 8,* 1-12.

Montgomery, D.C. (2005). *Introduction to Statistical Quality Control 5th Edition*. United States: John Wiley & Sons. 160 - 290.

Page, E.S. (1954). Continuous Inspection Schemes. *Biometrics*, *41*, 100-115.